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"Cosmological" Constant and Scalar Gravitons

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If a cosmological term is included in the equations of general relativity, the linearized equations can be interpreted as a tensor-scalar theory of finite-range gravitation. The scalar field cannot be transformed away by a gauge transformation (general co-ordinate transformation) and so must be interpreted as a physically significant degree of freedom. The hypothesis that a massive spin-two meson (mass m_2) satisfies equations identical in form to the equations of general relativity leads to the prediction of a massive spin-zero meson (mass m_0), the ratio of masses being $m_0/m_2=\sqrt{3}$.

§ 1. Introduction

Recent work by two of the authors[1] (K.P.S. and C.S.) has provided a very strong evidence that general relativity may play a crucial role in the physics of elementary particles, with the f-meson interpreted as a short-range gravitational field. In view of this, it is particularly important to understand the relationship between the gravitational theory with massless gravitons and the corresponding finite-range theory.

A characteristic of linear spin-s theories in special relativity is that the theories corresponding to zero mass are *not* in general limiting cases of the theories for particles with finite mass. For the mass zero cases, there are only two instead of $2s+1$ polarization states, and the free field equations are invariant under *gauge* transformations. The mass terms break the gauge invariance.

Boulware and Deser[2] have recently concluded that the gravitational theory of Einstein (which is a massless spin-2 theory) cannot also be obtained as the $m\to 0$ limit of a theory of finite-range gravity. However, these authors have imposed restrictions on the form of the mass term, which in our view are artificial restrictions leading to unnecessary difficulties. Conventional theory (without the cosmological term) is a theory of a massless spin-2 field. It is qualitatively different from the massless spin-2 theory of special relativity in two fundamentally important respects:

(i) It is highly nonlinear (self-interacting),

(ii) it is a geometrical theory. Space-time is no longer a passive background to the physics as in special-relativistic theories.

When a mass term is added in a linear spin-s theory, the gauge invariance is broken. There is no justification for the belief (based on analogy) that this is necessarily the case with a *nonlinear* spin-2 theory. Moreover, for finite-range gravity it would seem unreasonable to expect the gauge invariance to be broken, for the gauge invariance in this case is simply the requirement of covariance under general co-ordinate transformations. Boulware and Deser have argued that the mass term must break the gauge group in order to give rise to the extra degrees of freedom (extra spin states). This argument appears unconvincing for a non-linear theory when we observe that the concept of spin is only capable of precise definition when we go to the *linear approximation* (weak field approximation). Extra degrees of freedom will appear in the linearized theory provided the approximate (linearized) gauge group is violated by the approximate (linearized) equations—it is *not* necessary for the exact gauge transformations to be violated by an exact *non-linear* theory with non-zero rest mass.

When matter is absent, the flat (Minkowski) space-time is a solution of the massless gravitational theory. This enables the equations to be linearized when the field is weak and treated as special-relativistic equations with a flat background metric. Boulware and Deser require the mass term to be such that this solution is retained for the finite-range gravitational theory. However, the fact that space-time is experimentally nearly Minkowskian is adequately accounted for by requiring the range of the macroscopic gravitational theory to be sufficiently large compared to the size of regions in which space-time is known to be flat. The requirement of flatness of empty space-time has no more justification (on logical and experimental grounds) than the requirement of *uniform curvature*. Indeed, we could even argue that it is a *less* justifiable hypothesis, since flatness is a very special case of uniform curvature. We conjecture that the difficulties encountered by Boulware and Deser, and by Iwasaki[8] in tensor-scalar gravitational theories arise because the background space-time is taken to be flat.

Having argued that the mass-term in a finite-range version of Einstein's theory would not be expected to violate the gauge invariance (general covariance) and that the solution for completely empty space need not be Minkowskian, the obvious candidate for the mass term is the *cosmological term* of the usual theory.

§ 2. Spin-two theories

We review briefly the linear spin-2 theories in order to clarify their relationship with the non-linear theory of Einstein. For non-zero mass we have

$$\Box\phi_{\mu\nu}+m^2\phi_{\mu\nu}=0\,,\quad(\phi_{\mu\nu}=\phi_{\nu\mu}) \tag{2·1}$$

$$\partial_\mu\phi^{\mu\nu}=0\,,\ \ \phi_\mu{}^\mu=0\,. \tag{2·2}$$

These equations are completely equivalent to the Dirac-Fierz-Pauli spin-2 equations. The first subsidiary condition (2·2) eliminates four of the ten components of the wave function and the second eliminates one more, leaving $5=2s+1$ independent components.

The field of the massless spin-2 theory is a rank four tensor

$$\phi_{\mu\nu\rho\sigma}\,, \tag{2·3}$$

which has all the symmetries of a Weyl tensor[4] and satisfies

$$\partial_\mu\phi^{\mu\nu\rho\sigma}=0\,,\quad \partial_\lambda\phi_{\mu\nu\rho\sigma}+\partial_\mu\phi_{\nu\lambda\rho\sigma}+\partial_\nu\phi_{\lambda\mu\rho\sigma}=0\,. \tag{2·4}$$

The potentials $\zeta_{\mu\nu}=\zeta_{\nu\mu}$ are defined through

$$\phi_{\mu\nu\rho\sigma}=\partial_\mu\phi_{\nu\rho\sigma}-\partial_\nu\phi_{\mu\rho\sigma}\,,\quad \phi_{\rho\mu\nu}=\partial_\mu\zeta_{\nu\rho}-\partial_\nu\zeta_{\mu\rho}\,. \tag{2·5}$$

The second equation (2·4) is now automatically satisfied and the first is

$$\Box\zeta_{\mu\nu}+\partial_{\mu\nu}\zeta-\partial_{\nu\rho}\zeta_\mu{}^\rho-\partial_{\mu\rho}\zeta_\nu{}^\rho=0\,. \quad (\zeta=\zeta_\mu{}^\mu) \tag{2·6}$$

The gauge transformations

$$\zeta_{\mu\nu}\rightarrow\zeta_{\mu\nu}-\partial_\mu\xi_\nu-\partial_\nu\xi_\mu$$

leave the fields $\phi_{\mu\nu\rho\sigma}$ unchanged. The gauge group is restricted to transformations with the parameters satisfying

$$\Box\xi_\mu=0 \tag{2·7}$$

by a 'Lorentz condition'

$$\partial_\mu\zeta^{\mu\nu}-\tfrac{1}{2}\partial^\nu\zeta=0 \tag{2·8}$$

which reduces (2·6) to

$$\Box\zeta_{\mu\nu}=0\,. \tag{2·9}$$

The remaining gauge freedom can be used to transform away the trace (ζ) of the wave function.

§ 3. Linearized finite-range gravitation

The cosmological term in Einstein's equations

$$R_{\mu\nu}-\Lambda g_{\mu\nu}+\kappa(T_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}T)=0 \tag{3·1}$$

can be interpreted as a mass term for gravitons, giving rise to a finite range for the gravitational field. In the 'weak field approximation' we cannot regard the field as a small perturbation on a flat background metric, since even in the absence of matter the Minkowski metric is not a solution of (3·1). We can, however, take a space of constant curvature as a background. The appropriate metric is the de Sitter metric

$$e^{2\sigma}\eta_{\mu\nu}\,,\quad e^{-\sigma}=1-\frac{\Lambda}{12}x^2\,. \quad (x^2=x_\mu x^\mu=\eta_{\mu\nu}x^\mu x^\nu) \tag{3·2}$$

In the definition of σ and in the following discussion, indices are raised and lowered with the Minkowski metric $\eta_{\mu\nu}$. The Ricci tensor constructed from a metric $\eta_{\mu\nu}+\zeta_{\mu\nu}$ to first order in the $\zeta_{\mu\nu}$, is

$$\tfrac{1}{2}(\Box\zeta_{\mu\nu}+\zeta_{\cdot\mu\nu}-\zeta^{\rho}_{\mu\cdot\nu\rho}-\zeta^{\rho}_{\nu\cdot\mu\rho}),\quad (\zeta=\zeta_{\mu}{}^{\mu}) \tag{3·3}$$

Partial differentiation is denoted by a dot. That this has precisely the same form as the left-hand side of (2·6) is of course the justification for the statement that gravitions have spin 2. The gauge transformations are infinitesimal co-ordinate transformations. It is important to recognize that the interpretation of Einstein's theory as a theory of particles of spin-2 has meaning *only* in the linear approximation.

The Ricci tensor constructed from a metric

$$e^{2\sigma}(\eta_{\mu\nu}+\zeta_{\mu\nu}) \tag{3·4}$$

(representing small perturbations on a *de Sitter* background) can be obtained immediately from (3·3) by making use of the behaviour of a Ricci tensor under a conformal mapping.[5] We obtain

$$\begin{aligned} R_{\mu\nu}=\tfrac{1}{2}&(\Box\zeta_{\mu\nu}+\zeta_{\cdot\mu\nu}-\zeta^{\rho}_{\mu\cdot\nu\rho}-\zeta^{\rho}_{\nu\cdot\mu\rho}) \\ &+(\sigma_{\cdot\mu\nu}+\eta_{\mu\nu}\Box\sigma-2\sigma_{\cdot\mu}\sigma_{\cdot\nu}+2\eta_{\mu\nu}\sigma_{\cdot\rho}\sigma^{\cdot\rho}) \\ &-\sigma_{\cdot\rho}(\zeta^{\rho}_{\mu\cdot\nu}+\zeta^{\rho}_{\nu\cdot\mu}-\zeta^{\cdot\rho}_{\mu\nu}+\eta_{\mu\nu}[\zeta_{\lambda}{}^{\rho\cdot\lambda}-\tfrac{1}{2}\zeta^{\cdot\rho}+2\zeta_{\lambda}{}^{\rho}\sigma^{\cdot\lambda}]-2\zeta_{\mu\nu}\sigma^{\cdot\rho}) \\ &+(\zeta_{\mu\nu}\Box\sigma-\eta_{\mu\nu}\zeta^{\rho\lambda}\sigma_{\cdot\rho\lambda}). \end{aligned} \tag{3·5}$$

This is to be equated to

$$\Lambda e^{2\sigma}(\eta_{\mu\nu}+\zeta_{\mu\nu})-\kappa(T_{\mu\nu}-\tfrac{1}{2}\eta_{\mu\nu}T). \tag{3·6}$$

The second line of (3·5) is just the Ricci tensor of the background space, which by definition is equal to $\Lambda e^{2\sigma}\eta_{\mu\nu}$. Hence these terms in the equation can be deleted. We then have the linearized version of general relativity, but with a de Sitter background instead of a flat background.

The values of the derivatives of σ occurring in (3·5) are explicitly

$$\left.\begin{aligned} \sigma_{\cdot\mu}&=\frac{\Lambda}{6}e^{\sigma}x_{\mu}\,, \\ \sigma_{\cdot\mu\nu}&=\frac{\Lambda}{6}e^{\sigma}\left(\eta_{\mu\nu}+\frac{\Lambda}{6}e^{\sigma}x_{\mu}x_{\nu}\right), \\ \Box\sigma&=\frac{\Lambda}{6}e^{\sigma}\left(4+\frac{\Lambda}{6}e^{\sigma}x^{2}\right). \end{aligned}\right\} \tag{3·7}$$

The relationship between the perturbations $\zeta_{\mu\nu}$ and their derivatives at a chosen point of space-time, implied by the equality of (3·5) and (3·6), can be written down most simply by choosing the origin of the co-ordinate system to be the point in question. The σ-terms are simply replaced by their values at the origin

$$e^{\sigma}\sim 1,\ \sigma_{.\mu}\sim 0,\ \sigma_{.\mu\nu}\sim\frac{\Lambda}{6}\eta_{\mu\nu},\ \Box\sigma\sim\frac{2}{3}\Lambda. \tag{3·8}$$

The only terms that survive in (3·5) are the first line and the fourth line. At the origin, the equations become

$$\Box\zeta_{\mu\nu}-\beta_{\mu.\nu}-\beta_{\nu.\mu}\sim\tfrac{2}{3}\Lambda(\zeta_{\mu\nu}+\tfrac{1}{2}\eta_{\mu\nu}\zeta)-2\kappa(T_{\mu\nu}-\tfrac{1}{2}\eta_{\mu\nu}T), \tag{3·9}$$

$$\beta^{\mu}=(\zeta^{\mu\nu}-\tfrac{1}{2}\eta^{\mu\nu}\zeta)_{.\nu}\,. \tag{3·10}$$

The same equation as this was in fact obtained by Boulware and Deser, the peculiar form of the mass term (Λ-term) being derived by them from consistency requirements when a mass term is added to the linear massless spin-2 theory. That *precisely the same* mass term arises here from the linearization of Einstein's equations (3·1) is remarkable.

When the linearization is carried out on a flat background space-time, one usually imposes the (linearized) harmonic co-ordinate condition (cf. (2·8)):

$$\beta_{\mu}=0\,. \tag{3·11}$$

However, it is not valid to assume that this co-ordinate restriction is relevant in the present context. We could set β_{μ} equal to any quantity linear in the $\zeta_{\mu\nu}$ and containing first derivatives of σ in each term. This would have the desired effect of eliminating unwanted second derivatives of the $\zeta_{\mu\nu}$ that occur in conjunction with the d'Alembertian term in (3·5), but would give rise to additional Λ-terms in (3·9). The physical interpretation is therefore crucially dependent on the choice of subsidiary condition to replace (3·11), and we need a criterion for selecting the correct (physically significant) restriction. Moreover, we cannot assume that the $\zeta_{\mu\nu}$'s are the physical fields associated with 'gravitons'. In the usual linearization on a flat background the $\zeta_{\mu\nu}$ transform like a tensor under the Lorentz subgroup of the infinitesimal group of co-ordinate transformations. In the present case this is not so. The criterion we shall use for selecting the appropriate physical fields and the appropriate coordinate restriction, is the requirement of covariance under the de Sitter group.

§ 4. Gauge transformations

The gauge transformations for the theory given by the equality of (3·5) and (3·6) are the infinitesimal co-ordinate transformations:

$$x^{\mu}\to x^{\mu}+\xi^{\mu}\,. \tag{4·1}$$

By carrying out such a transformation on the metric (3·4) and putting the transformed tensor into the same form, we easily obtain the transformation laws (to first order in ξ^{μ})

$$e^{2\sigma}\to e^{2\sigma}(1+2\sigma_{.\rho}\xi^{\rho}), \tag{4·2}$$

$$\sigma_{\cdot\mu} \to (\sigma_{\cdot\mu} - \xi^{\rho}_{\cdot\mu}\sigma_{\cdot\rho}) - (\sigma_{\cdot\rho\mu}\xi^{\rho}), \tag{4·3}$$

$$e^{2\sigma}\zeta_{\mu\nu} \to e^{2\sigma}(\zeta_{\mu\nu} - \xi^{\rho}_{\cdot\mu}\zeta_{\rho\nu} - \xi^{\rho}_{\cdot\nu}\zeta_{\rho\mu}) - e^{2\sigma}(\xi_{\mu\cdot\nu} + \xi_{\nu\cdot\mu} + 2\eta_{\mu\nu}\sigma_{\cdot\rho}\xi^{\rho}), \tag{4·4}$$

The terms in the first bracket in (4·3) and (4·4) are the terms we would have if $\sigma_{\cdot\mu}$ and $e^{2\sigma}\zeta_{\mu\nu}$ were respectively a vector and a tensor. The second bracket in (4·4) brings out the analogy between the behaviour of $e^{2\sigma}\zeta_{\mu\nu}$ under infinitesimal co-ordinate transformations and the behaviour of the four-potential under electromagnetic gauge transformations. Defining the quantities $\chi^{\mu\nu}$ from the contravariant tensor density

$$\sqrt{-g}\, g^{\mu\nu} = e^{2\sigma}(\eta^{\mu\nu} - \chi^{\mu\nu}), \tag{4·5}$$

we find that the transformation law (to first order in ξ^{μ}) is

$$\left.\begin{aligned} e^{2\sigma}\chi^{\mu\nu} &\to e^{2\sigma}(\chi^{\mu\nu} + \xi^{\mu}_{\cdot\rho}\chi^{\rho\nu} + \xi^{\nu}_{\cdot\rho}\chi^{\rho\mu} - \xi^{\rho}_{\cdot\rho}\chi^{\mu\nu}) \\ &\quad - e^{2\sigma}(\xi^{\mu\cdot\nu} + \xi^{\nu\cdot\mu} - \eta^{\mu\nu}\xi^{\rho}_{\cdot\rho} - 2\eta^{\mu\nu}\sigma_{\cdot\rho}\xi^{\rho}). \end{aligned}\right\} \tag{4·6}$$

The homogeneous terms (first bracket) are the terms we would have if $e^{2\sigma}\chi^{\mu\nu}$ were a tensor density (of weight 1).

The inhomogeneous terms in (4·4) (and also in (4·6)) vanish if and only if the parameters ξ^{μ} have the form

$$\xi^{\mu} = \lambda^{\mu} - \frac{A}{6}\left(\lambda^{\rho}x_{\rho}x^{\mu} - \frac{1}{2}\lambda^{\mu}x^{2}\right) + \lambda^{\mu\nu}x_{\nu}, \tag{4·7}$$

where λ^{μ} and $\lambda^{\mu\nu} = -\lambda^{\nu\mu}$ are constants. Equation (4·7) is the general solution of $\xi_{\mu\cdot\nu} + \xi_{\nu\cdot\mu} = -2\eta_{\mu\nu}\sigma_{\cdot\rho}\xi^{\rho}$. This corresponds to the infinitesimal *de Sitter group*. The quantities $e^{2\sigma}\zeta_{\mu\nu}$ transform like a tensor under the de Sitter group, and $e^{2\sigma}\chi^{\mu\nu}$ transform like a tensor density.

From the quantities $e^{2\sigma}\zeta_{\mu\nu}$ we can form a de Sitter *invariant*

$$\phi = g^{\mu\nu}e^{2\sigma}\zeta_{\mu\nu} = \eta^{\mu\nu}\zeta_{\mu\nu} = \zeta \tag{4·8}$$

and a set of quantities that transform like a *traceless tensor* under the de Sitter group.

$$\phi_{\mu\nu} = e^{2\sigma}\zeta_{\mu\nu} - \tfrac{1}{4}g_{\mu\nu}\phi = e^{2\sigma}(\zeta_{\mu\nu} - \tfrac{1}{4}\eta_{\mu\nu}\zeta). \tag{4·9}$$

We shall interpret ϕ and $\phi_{\mu\nu}$ as the fields (wave-functions) associated with a spin-zero particle and a spin-2 particle respectively.

A de Sitter covariant restriction on the fields is provided by the requirement that the *covariant* divergence of the tensor density $e^{2\sigma}\chi^{\mu\nu}$ shall be zero:

$$(e^{2\sigma}\chi^{\mu\nu})_{\cdot\mu} + \left\{ {\nu \atop \rho\mu} \right\} e^{2\sigma}\chi^{\rho\mu} = 0, \tag{4·10}$$

where $\left\{ {\nu \atop \rho\mu} \right\}$ are the Christoffel symbols constructed from the de Sitter metric (3·2). To first order in the $\zeta_{\mu\nu}$, $\chi^{\mu\nu} = \zeta^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}\zeta$ so this restriction is

$$\beta^{\nu} = \sigma^{\cdot\nu}\zeta - 4\sigma_{\cdot\mu}\zeta^{\mu\nu}. \tag{4·11}$$

Equation (4·11) is the *de Sitter covariant* version of (3·11). Rewriting (3·9) in terms of the de Sitter covariant fields $\phi_{\mu\nu}$ and ϕ and imposing (4·11) lead to the relations

$$\left.\begin{aligned}&(\Box-2\Lambda)\phi\sim 2\kappa T\,,\\&(\Box-\tfrac{2}{3}\Lambda)\phi_{\mu\nu}\sim -2\kappa(T_{\mu\nu}-\tfrac{1}{4}\eta_{\mu\nu}T)\,.\end{aligned}\right\}\qquad(4\cdot 12)$$

If Λ is *negative* these are in the form of Klein-Gordon equations for a massive spin-zero field and a massive spin-2 field, with source terms, with Compton wavelengths $m_0c/\hbar=\sqrt{-2\Lambda}$ and $m_2c/\hbar=\sqrt{-2\Lambda/3}$. Spin-one is eliminated by the subsidiary condition. Note that Eqs. (4·11) serve only to identify the masses of the particles—they are relations satisfied at a single point, and not true field equations—the field equations are the extremely complicated (3.5, 6). The ratio of masses predicted for the two kinds of 'heavy graviton' on the basis of this theory is

$$\frac{m_0}{m_2}=\sqrt{3}\,.\qquad(4\cdot 13)$$

§ 5. Geometrical and physical interpretation

The Einstein equations with $\Lambda=0$, when linearized on a flat background metric, become the equations of a Lorentz-covariant theory of a massless spin-2 particle. We have shown that with $\Lambda<0$ the equations linearized on a de Sitter background become the equations of a *de Sitter covariant* theory of massive spin-2 *and spin-zero* particles. The equations can easily be expressed in a form in which the covariance under de Sitter transformations is readily apparent.

In the following, indices are raised and lowered with the de Sitter metric (3·1) and the notation $|\mu$ is used for covariant derivatives constructed from the Christoffel symbols associated with that metric. We easily find that, at the origin,

$$\phi|_{\rho}^{\rho}\sim\Box\phi\,,\quad \phi_{\mu\nu}|_{\rho}^{\rho}\sim\Box\phi_{\mu\nu}-\frac{4}{3}\Lambda\phi_{\mu\nu}\,.\qquad(5\cdot 1)$$

Then (4·12) is

$$\left.\begin{aligned}&\phi|_{\rho}^{\rho}-2\Lambda\sim 2\kappa T\,,\\&\phi_{\mu\nu}|_{\rho}^{\rho}+\tfrac{2}{3}\Lambda\phi_{\mu\nu}\sim -2\kappa(T_{\mu\nu}-\tfrac{1}{4}g_{\mu\nu}T)\,.\ \ (T=g^{\mu\nu}T_{\mu\nu})\end{aligned}\right\}\qquad(5\cdot 2)$$

These are *tensor equations* (i.e., both sides transform covariantly under de Sitter transformations) satisfied at the origin. Since any point can be chosen as the origin by carrying out a de Sitter transformation, we can infer the validity of the equations everywhere. The Einstein equations linearized on a de Sitter background are equivalent to the de Sitter covariant equations

$$\left.\begin{aligned}&\phi|^{\rho}_{\rho}-2\Lambda\phi=2\kappa T\,,\\&\phi_{\mu\nu}|^{\rho}_{\rho}+\tfrac{2}{3}\Lambda\phi_{\mu\nu}=-2\kappa(T_{\mu\nu}-\tfrac{1}{4}g_{\mu\nu}T),\ \ (\phi_{\mu\nu}g^{\mu\nu}=0)\end{aligned}\right\} \tag{5·3}$$

provided we impose the (de Sitter covariant) subsidiary condition

$$\psi^{\mu\nu}_{|\mu}=0\,,\ \psi^{\mu\nu}=g^{\mu\rho}g^{\nu\lambda}(\phi_{\rho\lambda}-\tfrac{1}{4}g_{\rho\lambda}\phi). \tag{5·4}$$

From (5·3) and (5·4) we can easily deduce the continuity equation

$$T^{\mu\nu}_{|\mu}=0\,. \tag{5·5}$$

To obtain this we must bear in mind that the derivatives do not commute but have a commutation law defined in terms of the Riemann tensor of de Sitter space, which is

$$-\frac{\Lambda}{3}(g_{\mu\rho}g_{\nu\lambda}-g_{\mu\lambda}g_{\nu\rho})\,. \tag{5·6}$$

Equation (5·5) follows from the massive tensor-scalar theory given by (5·3) and (5·4) *only if* the Λ terms are precisely the ones indicated, corresponding to mass ratio $\sqrt{3}$.

The usual linearized massless gravitational theory is obtained as the limiting case ($\Lambda\to 0$) of the theory given by (5·3) and (5·4). The massive linearized theory is not invariant under general gauge transformation (infinitesimal co-ordinate transformations), but it is invariant under the de Sitter subgroup. In the limit $\Lambda\to 0$, the background space-time becomes Minkowskian and the equations become

$$\Box\zeta_{\mu\nu}=-2\kappa(T_{\mu\nu}-\tfrac{1}{2}\eta_{\mu\nu}T) \tag{5·7}$$

with a subsidiary condition

$$\zeta^{\mu\nu}_{\cdot\nu}=\tfrac{1}{2}\zeta^{\cdot\mu} \tag{5·8}$$

corresponding to the usual linearized Einstein equations with the harmonic co-ordinate condition. In the limit the full gauge-invariance is restored, except that the condition (5·8) imposes the restriction $\Box\xi^{\mu}=0$ on the gauge parameters. The usual linearized Einstein theory *with harmonic co-ordinate condition* is a tensor-scalar theory, and not a pure spin-2 theory. Only when the energy-momentum tensor is *traceless* can we find an infinitesimal co-ordinate transformation that respects the harmonic condition, and that transforms away the scalar. We then obtain a pure massless spin-2 theory.

We feel that the importance of the finite-range tensor-scalar gravitational theory lies not so much in the possibility of interpretation as a gravitational theory in the usual (macroscopic) sense (where Λ is very small and possibly zero), but rather in applications to particle physics. The hypothesis here is that the strong interactions take place in regions of intensely curved space-time.[1] In these regions the coupling constants G of the gravitational theory is replaced by a constant of the same order as the strong coupling constants, and the relevant 'cosmological' constant is $-\frac{3}{2}m_f^2$ (suggesting a very short range $\sim 10^{-14}$ cm). The f-meson and a scalar meson of mass $m_f\sqrt{3}$ together determine the metric, through equations *identical in form* with Einstein's equations (3·1). The relationship between the mesons and massless gravitons is analogous to the well-known behaviour of ρ^0 as a 'massive photon'.[5]

Chen[7] has shown that, if a mass term is included in the equations of a (spin-1) Yang-Mills field, the invariance under the Yang-Mills group can be retained in the presence of a mass term by introducing a spin-zero field into the formalism. The spin-zero field in 'finite-range gravity' plays precisely this role. In this context it may be noted that Utiyama[8] was the first to point out that the gravitational field of Einstein's theory is a kind of non-abelian gauge-field for the general co-ordinate transformation (see also the work of Kibble,[9] Sciama[10] and Lord.[11]) Equation (3·1) in fact represents this Yang-Mills (i.e., non-abelian gauge field) theory with a mass term that does not violate this gauge group.

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