Strengthening Mechanisms
Design Principle

Increase the intrinsic resistance to dislocation motion.

Generally, ductility suffers when strength increases!
**Possible Ways**

Dislocation interaction with

1) other dislocations -- strain hardening
2) grain boundaries -- grain boundary strengthening
3) solute atoms -- solid solution strengthening
4) precipitates -- precipitation hardening
5) dispersoids -- dispersion strengthening
Strain (Work) Hardening

Diagram showing the relationship between stress, percent cold work, and strain.
Industrial Importance
Strength vs. Dislocation Density

![Graph showing the relationship between strength and dislocation density.](image)
Stress-strain response of a FCC crystal oriented for single slip

I: Easy glide (primary slip)

\[ \theta_1 \sim G/30 \]
Characterized by long (100 to 1000 \( \mu \)m), straight and uniformly Spaced (10 to 100 nm apart) slip lines

II: Linear hardening (secondary slip)

\[ \theta_{II} \sim G/300 \]

III: Parabolic hardening (dynamic recovery & Cross-slip)

Resembles stress-strain response of a polycrystalline form of the same metal. Onset of stage III depends greatly on SFE and temp.

The extent of three stages depends on test T, purity, initial dislocation density, and orientation.
Work Hardening Theories

• Several theories, all focused on dislocation interaction mechanisms.
• Taylor (1934) first recognized that work hardening is due to dislocation interactions.
• Seeger & Friedel: Dislocation pileups at obstacles such as Lomer-Cottrell locks. Explains stage II hardening well.
• Kuhlmann-Wilsdorf: Mesh length theory based on dislocation cells. Comprehensive.
Work-hardening Theories

• Recall Orowan Equation: $\gamma_p = k\rho b\bar{l}$

• Difficult to predict work-hardening behavior because of unpredictability of the strain which is a function of both $\rho$ and distribution.

• Stress is a state function whereas plastic strain is not (depends on the history).

• Model processes by which lead to different dislocation configurations and correlate with experimental observations.
Taylor’s Theory (1934)

1) Moving dislocations interact with each other elastically and get trapped.

2) Trapped dislocations give rise to internal stresses that increase the stress necessary for deformation.

3) Only considered edge dislocations and assumed uniform distribution.

$$\Rightarrow \text{Spacing between dislocations, } L = \rho^{-0.5}$$

4) The effective internal stress, $\tau$, caused by these interactions is the stress necessary to force two dislocations past each other.
Consider a simple case of an edge dislocation moving from A to B. Minimum approach distance of other dislocations is $L/2$. 
• Considering the repulsive force, the shear stress is

\[ \sigma_{12} = \frac{Gb}{2\pi(1-v)} \frac{x_1(x_1^2 - x_2^2)}{(x_1^2 + x_2^2)^2} \]

• Supposing that \( x_1 = L/2 \) and \( x_2 = 0 \)

\[ \sigma_{12} = \frac{Gb}{\pi(1-v)L} = \frac{Kb}{L} \]

• In order to overcome \( \sigma_{12} \), a shear stress \( \tau = \sigma_{12} \) has to be applied.

\[ \tau = Kb\sqrt{\rho} \]
• Orowan’s equation: $\gamma = k \rho b l$

• Combining we get,

$$\tau = K b \sqrt{\frac{\gamma}{k b l}} = k' \sqrt{\gamma}$$

• Add a frictional term, $\tau_0$ (stress required to move a disln. in the absence of other dislns.)

$$\tau = \tau_0 + k' \sqrt{\gamma}$$

• Describes the behavior of many materials at large strains.
Drawbacks

• Regular configurations of dislocations rarely observed.
• Screw dislocations are not involved, hence cross-slip is not considered.
• Dislocations on different planes can trap each other and may not be able to move independently.
• Doesn't explain linear hardening (Stage II)
• Deformation tends to be nonuniform.
Kuhlmann-Wilsdorf Theory

- Doesn’t depend on a specific disln. model
- Stage I: A heterogeneous distribution of dislns. exists. They can move easily and hence low work hardening rate, $\theta_1$.
- Stage I ends when a fairly uniform disln. distribution of moderate density. The existence of a quasi-uniform array with clusters of dislocations surrounding cells of relatively low dislocation density.
- Represents minimum energy, hence preferred configuration.
• High SFE--Narrow cell walls, low $\rho$ in cells
• Low SFE--Dislocation planar arrays (because cross-slip is difficult).
• Stress necessary for further plastic deformation depends on mean free dislocation path, $\bar{l}$

(a la Frank - Read Source) $\tau \propto Gb / \bar{l}$

• Since $\rho \propto 1 / \bar{l}^2$, $\Delta \tau \propto Gb \sqrt{\rho}$
• With increasing plastic deformation, $\rho \uparrow$ and hence $\bar{l} \downarrow \Rightarrow \Delta \tau \uparrow$
• Note that *character* of dislocation distribution remains unchanged, only the *scale* of the distribution changes.

• Continued reduction of cell size in stage II.

Variation of dislocation cell size with %CW in polycrystalline Nb-Steel alloy
• Eventually, a point is reached where $\bar{l}$ reaches a steady state value. Onset of stage III.

• Cross-slip gets activated. Hence, the onset of stage III is sensitive to SFE. Higher the SFE, sooner (lower stress) will be the onset!

• Also, higher SFE can help in accelerating the dislocation rearrangement process and hence lower stress levels for the stabilization of $\bar{l}$.
Single- vs. Poly-Crystal Behavior

- Qualitatively, the tensile stress-plastic strain response of polycrystal resembles stage III in a single crystal of the same material.
- Relationship: \( \sigma = \tau M \) where \( M = 1/\cos \phi \cos \lambda \) (1)
- Assuming no texture in the polycrystalline material, replace \( M \) with \( \overline{M} \)
- Difficult to compute \( \overline{M} \) (384 combinations of 5 independent slip systems)
- Taylor postulated that the preferred combination is that which sum of glide shears are minimized.
\[ \varepsilon = \gamma / \overline{M} \quad (2) \]

- Combining (1) and (2) we get

\[ \frac{d\sigma}{d\varepsilon} = \overline{M}^2 \frac{d\tau}{d\gamma} \]

- For the case of \{111\}<110> in FCC and \{110\}<111> in BCC, \( \overline{M} = 3.07 \)

\[ \Rightarrow \text{The strain hardening rate in polycrystals is much larger than that seen in single crystals.} \]
Grain-Boundary Strengthening

- $T < 0.5T_m$: GBs act as barriers to dislocation motion.
- Compatibility of deformation in neighboring grains necessitates multiple slip (the operation of 5 independent slip systems) and hence high strain hardening rate.
- $T > 0.5T_m$: GBs provide for higher rate diffusion paths. Results in faster creep.
- At high temps. Grain boundary sliding is another possible mechanism for enhanced creep.
- GBs lead to texture and associated affects.
The Hall-Petch Relationship

\[ \sigma_y = \sigma_0 + kD^{-0.5} \]
Dislocation arrays in AISI 304 steel ($\varepsilon = 1.5\%$)
The $\sigma_y \propto D^{-0.5}$ is observed to be true in a no. of metals.
GB hardening is valid only for $D \sim 10-100 \mu m$
Hall-Petch Theory (1951)

• Leading dislocation in the pile-up bursts through a GB due to stress concentration.

• The no. of dislocations, \( n \), that can occupy distance, \( d \) (\( d = D/2 \)) between the dislocation source and the GB is given by Eshelby et al. (1951)

\[
n = \frac{\pi \alpha \tau_s d}{Gb}
\]

where \( \tau_s \)-resolved shear stress
\( \alpha \)-constant = 1 for screw dislocations
\( = (1-v) \) for edge dislocations
• The stress on the leading dislocation = $n\tau_s$

• When $n\tau_s > \tau_c$, a critical resolved stress, the dislocation will be able to burst through.

$$\frac{\pi \alpha D \tau_s}{2Gb} \tau_s \geq \tau_c$$

• To account for the frictional stress to move the dislocation, we add $\tau_0$ term.

$$\tau_s \geq \tau_0 + kD^{-1/2}$$

• Valid only for larger D as the Eshelby’s equation is valid only for large dislocation arrays
Cottrell’s Theory (1958)

- Recognized that it is virtually impossible for dislocations to burst through.
- Instead considers stress concentration caused by a pile-up in one grain activating a dislocation source in a neighboring grain.
• Treated the pile-up as a shear crack.
• The maximum shear stress at a distance $r$ ahead of the crack-tip

$$\tau = (\tau_s - \tau_0) \left(\frac{D}{4r}\right)^{1/2}$$

• When $\tau = \tau_c$, F-R source gets activated.
• Rearranging, we get

$$\tau_s = \tau_0 + 4\tau_c r^{1/2} D^{-1/2}$$
Other Theories

• More recent theories suggest that grain boundaries as a source of dislocations.
• Onset of yielding occurs when the GB dislocation sources get activated.
• Such GB sources act as forests and yield stress depends on the stress required to move dislocations through these forests.
• Li’s theory (1963): \( \tau = \tau_0 + \alpha G b \sqrt{\rho} \)
• Dislocation Density, \( \rho \propto 1/D \)
Solid Solution Strengthening

• A result of the elastic interaction between the stress fields of dislocations and solute atoms.
• Recall that the stress field of an edge dislocation has both shear and hydrostatic components whereas a screw dislocation has only shear components.
• Shear stress field surrounding a screw dislocation is distortional whereas in an edge, it is both distortional and dilatational.
\[ |\tau| = |\sigma_t| = |\sigma_c| \]
Interaction Energy, $E_I$

If $E_I < 0$, work $|E_I|$ is required to separate dislocation from the point defect.

Elastic sphere of radius $r_a(1+\delta)$ and vol. $V_s$ is inserted into a spherical hole of radius $r_a$ and vol. $V_h$ in an elastic matrix. Both the sphere and the matrix are isotropic with the same $G$ and $\nu$. 
• Misfit volume, $V_{\text{mis}} = V_s - V_h$
  \[ \approx 4\pi r_a^3 \delta \text{ (if } \delta \ll 1) \]

• Misfit parameter $\delta$ is $+ve$ for oversized defects and $-ve$ for undersized defects.

• On inserting the sphere in the hole, $V_h$ changes by $\Delta V_h$ to leave a final defect of radius $r_a(1+\varepsilon)$.

\[ \Delta V_h = \frac{4}{3} \pi r_a^3 (1 + \varepsilon)^3 - \frac{4}{3} \pi r_a^3 \]

\[ \approx 4\pi r_a^3 \varepsilon \text{ (for } \varepsilon \ll 1) \]
• Parameter $\varepsilon$ is determined by equilibrium at the interface (Eshelby 1956 and 1957).

$$\Delta V_h = \frac{(1+\nu)}{3(1-\nu)} V_{mis} \quad \text{i.e. } \quad \varepsilon = \frac{(1+\nu)}{3(1-\nu)} \delta$$

• The total volume change for an infinite matrix is $\Delta V_h$. In a finite body, traction-free BCs results in a volume change, $\Delta V$

$$\Delta V = \frac{3(1-\nu)}{(1+\nu)} \Delta V_h = V_{mis}$$
• The strain energy change due to the presence of point defect when the material is subjected to pressure $p$, $E_I = p\Delta V$
• For a dislocation, $p$ is evaluated at the site of the defect.
• For screw dislocation, $p = 0$ and hence $E_I = 0$.
• For an Edge dislocation,

$$E_I = \frac{4(1+\nu)}{3(1-\nu)} G b r_a^3 \delta \frac{\sin \theta}{r}$$
• For $\delta>0$ (oversized defect), $E_I$ is positive for sites above the slip plane ($0<\theta<\pi$) and negative for sites below the slip plane ($\pi<\theta<2\pi$).

$$E_I = \frac{4(1+\nu)}{3(1-\nu)} Gbr_\alpha^3 \delta \frac{\sin \theta}{r}$$
• For $\delta<0$ (undersized defect), $E_1$ is positive for sites below the slip plane ($\pi<\theta<2\pi$) and negative for sites above the slip plane.
• For a given species of defects, the $\delta$ can be determined by measuring the lattice parameter (and in turn lattice strain) as a function of defect concentration.
• Vacancies: $\delta$ varies between -0.1 to 0.
• Substitutional solutes: $\delta$ varies between -0.15 to +0.15
• Interstitial atoms: $\delta$ varies between -0.1 to +1
• Upper bound of interaction energy, $E_I$, varies between $3 |\delta|$ (for close packed metals) to $20 |\delta|$ (for Si and Ge).
Asymmetrical Defects

- Many defects occupy lower symmetry sites and hence produce asymmetric distortions.
- Interact with both hydrostatic and shear components of the stress field.
- Therefore, interact with both edge and screw dislocations.
- Example: C in BCC Fe, which occupies the octahedral site.
Carbon in BCC Iron

- Atoms E & F are $a/2$ distance away.
- Atoms A, B, C, & D are $a/\sqrt{2}$ away.
- Interstitial atom produces a tetragonal distortion.

Misfits, $\delta_{xx} = \delta_{yy} = -0.05; \delta_{zz} = +0.43$

$E_I$ for both edge and screw dislns. are comparable
Asymmetrical defects result in higher rate of strengthening
Defects with different Elastic Constants

• Vacancy, a soft region with zero modulus.
• Point defects can increase or decrease the modulus of the surrounding matrix.
• Both soft and hard defects induce a change in the stress field of a dislocation, leading to inhomogeneity interaction energy.
• Attractive for soft defects (analogous to attraction of dislocations to free surfaces). Repulsive for hard defects.
• Proportional to $1/r^2$. Important when $\delta$ is small.
Other types of Interactions

• *Electrical interaction* between electron density dipoles associated with dislocations with those of solute atoms having different valence. Negligible.

• *Suzuki effect*: Stacking faults have different solute solubility. The resulting change in chemical potential will cause solute atoms to diffuse to the fault, acting as barriers to dislocations.
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<th>Material</th>
<th>Defect</th>
<th>Hardening Effect</th>
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<td></td>
<td></td>
</tr>
<tr>
<td>Al</td>
<td>Substitutional atom</td>
<td>$G/10$</td>
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<tr>
<td>Cu</td>
<td>Substitutional atom</td>
<td>$G/20$</td>
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<td>Fe</td>
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<td>Nb</td>
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<tr>
<td>NaCl</td>
<td>Monovalent substitutional ion</td>
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<tr>
<td><strong>Nonsymmetrical Defects</strong></td>
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<td>Al</td>
<td>Vacancy disk (quenched)</td>
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<tr>
<td>Cu</td>
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<td>Fe</td>
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<td>LiF</td>
<td>Interstitial fluorine (irradiation)</td>
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</tr>
<tr>
<td>NaCl</td>
<td>Divalent substitutional ion</td>
<td>$2G$</td>
</tr>
</tbody>
</table>
Yield Point Phenomenon
Cottrell Atmospheres
• Solute atoms diffuse to dislocations and form atmospheres around dislocation cores, pinning them.
• High stress is needed to rip the dislocation through these atmospheres.
• Unpinned dislocations multiply rapidly by multiple-cross-slip mechanism, no. of mobile dislocations increase rapidly, yielding becomes easier, and stress necessary for further plastic deformation drops.
• Localized bands (Lüder bands).
• Once all dislocations have broken free, homogeneous plastic flow starts.
Low-carbon steel in a temper-rolled condition & annealed for one hour between 100 and 343°C
Portevin-Le Chatelier Effect

• Dynamic strain aging or serrated flow
• Occurs at high deformation temperatures and low strain rates.
• Result of high mobility of point defects, which diffuse back to the dislocation cores because of the low strain rates applied.
Precipitation Hardening

![Phase diagram showing precipitation hardening](image)

Temperature, °C

% copper

Liquid

α + liquid

Homogeneous solid solution

α + CuAl₂ precipitate
Heat Treatment Procedure

Solution heat treatment

Quench

Precipitation heat treatment

Temperature

$T_0$

$T_2$

$T_1$

Time
Property Change with aging Time
Aging curves for 6061-T4 alloy
Three Stages during Aging

- **Stage I**: Clustering of solute atoms (Guinier-Preston zones). Incubation period. The G-P zones are coherent with the matrix. (under-aged)
- **Stage II**: Nucleation and growth of second phase particles, until an equilibrium precipitate volume fraction is reached (peak-aged).
- **Stage III** (Ostwald ripening): Coarsening of precipitates, with large particles growing at the expense of small ones. (over-aged)
As-solutionized  

G-P zones  

Incoherent Precipitates
Coherent Precipitates

- Dislocations cut through the precipitate.
- Lattice misfit leads to elastic strain fields surrounding the precipitate.
- Shape of the precipitate depends on the misfit. Small misfit, spherical particles. Increasing misfit leads to cuboids (moderate misfit), aligned cubes and rod-like particles.
- Also depends on, anisotropy in interfacial energy as well as anisotropy in misfit.
- Growth kinetics also influence the shape.
Completely coherent

Strained but Coherent

Semi-coherent

Incoherent
G-P zones in Al-16% Ag Alloy
Precipitates in (a) Al-Li alloy and (b) Ni-Al alloy
Precipitates in a Al-Cu alloy
Grain Boundary Precipitates and associated precipitate-free zone in an Al-Li alloy
Cuboid (Ni$_3$Al) and Carbide Precipitates in a Ni-base Superalloy
Dislocation-Particle Interactions

• Strengthening depends on whether a dislocation cuts through or loops around a precipitate.
• Misfit strain strengthening:
  \[ \tau \propto G\varepsilon^{3/2} (rf)^{1/2} \]
  where \( \varepsilon \) is misfit strain (\( \propto \) lattice parameter mismatch)
  \( r \) is particle radius
  \( f \) is volume fraction of precipitates.
• Strengthening contribution is relatively small
• Energy storing mechanisms with the generation of new interphase boundary and APB energy.
• Interphase boundary energy of coherent precipitates is small, hence contribution is small.
• APB energy is $\sim 10$ times larger than the interphase boundary. Hence contributes significantly.

$$\tau \propto \gamma^{3/2} \left( \frac{rf}{G} \right)^{1/2}$$

where $\gamma$ is APB energy
• Passage of superlattice dislocation pairs reduces the length of ordered path for subsequent dislocations. It becomes easier for them to move on the same plane. Results in planar slip and large slip steps. Leads to loss of ductility in certain alloys (e.g. Al-Li alloys.)

• The P-N stress for dislocation to move through the precipitate can also be higher.
Orowan Looping

- Occurs when it is easier for the dislocation to bypass a precipitate by looping around it than cutting through it.
• Recall: Stress, $\tau$, necessary to bend a dislocation to a radius, $r$ is given by $\tau \sim \frac{Gb}{2r}$
• If $l$ is the separation between two particles in a slip plane, the dislocation must be bent to a radius $l/2$ for the dislocation to loop around.
• Thus, $\tau \sim \frac{Gb}{l}$ (strictly speaking $\tau \sim \frac{2T}{bl}$)
• With increasing dislocation loops, $l$ decreases and hence it becomes difficult to further loop. More stress is required. Work hardening.
• For fixed $f$, $l$ increases with aging time.