High Strain Rate Experiments

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Topics Covered

- Introduction to High Strain Rate Experiments
- Review of One Dimensional Elastodynamics
- Kolsky Bar Experiments - Compression, Tension, and Torsion
- Kolsky Bar Experimental Setup
- Wave Propagation Analysis - Dynamic Material Property Extraction
  1. Compression and Tension
  2. Torsion
  3. Adiabatic Heating
- Important Features of Kolsky Bar Experimental Technique
  1. Specimen Geometry - Friction, Homogenization, Inertial Effects
  2. Specimen Material (Metal/Ceramic/Polymer) - Pulse Shaping
  3. Wave Dispersion
  4. High/Low Temperature Experiments
High Strain Rate Experiments on Materials

- Dynamic loading causes significant deformation and damage of materials due to the kinetic energy carried by a mass as compared to quasi-static loading.
- In dynamic loading stress is a function of the particle velocity, i.e., the motion of an individual point mass in the body.
- Upon impact of a body a stress wave is generated, which travels at a specified velocity.
- The velocity of the stress wave is different from the velocity of the particle in motion.
- Stress wave velocity depends on the nature of the wave and the material properties, such as, density and modulus.
- Dynamic deformation of materials involves stress wave propagation.
- High strain rate experiments are performed to extract constitutive behavior, failure, fracture and fragmentation of materials.
High Strain Rate Experimental Techniques

- Quasi-static strain rates - $< 10^{-3}\text{s}^{-1}$
- Intermediate strain rates - $> 10^{-3}\text{s}^{-1}$
- High strain rates - $> 10^{2}\text{s}^{-1}$
- Very high strain rates - $> 10^{4}\text{s}^{-1}$
- Ultra high strain rates - $> 10^{6}\text{s}^{-1}$

The choice of experimental technique determines the strain rate range accessible for different materials.

Material properties that affect that the strain rate include, density, wave speed, yield strength, etc.

The specimen geometry also changes between the techniques to ensure that uniform stress state exists during the experiment.

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One Dimensional Wave Propagation in Elastic Media

From Linear Elasticity Theory:

**Equilibrium (elasto-statics)**

\[ \sigma_{ij,j} + b_i = 0 \quad (\Sigma F = 0) \]

**Equilibrium (elasto-dynamics)**

\[ \sigma_{ij,j} + b_i = \rho \ddot{u}_i \quad (\Sigma F = ma) \]

- \( \sigma_{ij} \) - stress component
- \( b_i \) - body force component
- \( u_i \) - displacement component
- \( \rho \) - material density
One Dimensional Wave Propagation in Elastic Media

1-D equilibrium in x-direction without body forces:

\[ \sigma_{xx,x} = \rho \ddot{u}_x \quad (1) \]

1-D linear elastic material:

\[ \sigma_{xx} = E \varepsilon_{xx} \quad (2) \]

From (1) and (2)

\[ E \varepsilon_{xx,x} = \rho \ddot{u}_x \quad (3) \]

\[ c_0^2 \frac{\partial^2 u_x}{\partial x^2} = \frac{\partial^2 u_x}{\partial t^2} \quad (4) \]

where \( c_0 = \sqrt{\frac{E}{\rho}} \) is the "longitudinal wave speed"
One Dimensional Wave Propagation in Elastic Media

General Solution for the 1-D wave equation

\[ c_0^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0 \]

This is a hyperbolic equation and there are at least two methods to solve this PDE, separation of variables and method of characteristics.

Using either one of the methods it can be shown that the general solution is of the form:

\[ u(x, t) = F(x - c_0 t) + G(x + c_0 t) \]

where, F and G are harmonic functions.

F is the Right Traveling Wave (RTW) and G is the Left Traveling Wave (LTW)
One Dimensional Wave Propagation in Elastic Media

**Right Traveling Wave** - \( F(x - c_0 t) \). At any time \( t > 0; x = c_0 t \)

\[
\begin{align*}
  u(x, t) &= F(x - c_0 t) \\
  \frac{\partial u}{\partial t} &= -c_0 F'(x - c_0 t); \\
  \frac{\partial u}{\partial x} &= F'(x - c_0 t) \\
  \sigma(x, t) &= \varepsilon_{xx} = E \frac{\partial u}{\partial x} = EF'(x - c_0 t)
\end{align*}
\]

So,

\[
\sigma(x, t) = -\frac{E}{c_0} \frac{\partial u}{\partial t} = -\rho c_0 \frac{\partial u}{\partial t}
\]

\( \dot{u} = \frac{\partial u}{\partial t} \) - is the particle velocity

\(-\rho c_0 \) - is the acoustic impedance of the material

if \( \dot{u} = \frac{\partial u}{\partial t} > 0 \Rightarrow \sigma < 0 \) (Compression)

if \( \frac{\partial u}{\partial t} < 0 \Rightarrow \sigma > 0 \) (Tension)
One Dimensional Wave Propagation in Elastic Media

Left Traveling Wave - \( G(x + c_0 t) \). At any time \( t > 0; x = -c_0 t \)

\[
\begin{align*}
    u(x, t) &= G(x + c_0 t) \\
    \frac{\partial u}{\partial t} &= c_0 G'(x + c_0 t); \quad \frac{\partial u}{\partial x} = G'(x + c_0 t) \\
    \sigma(x, t) &= E \varepsilon_{xx} = E \frac{\partial u}{\partial x} = EG'(x + c_0 t)
\end{align*}
\]

So,

\[
\begin{align*}
    \sigma(x, t) &= \frac{E}{c_0} \frac{\partial u}{\partial t} = \rho c_0 \frac{\partial u}{\partial t}
\end{align*}
\]

if \( \dot{u} = \frac{\partial u}{\partial t} > 0 \Rightarrow \sigma > 0 \) (Tension)

if \( \dot{u} = \frac{\partial u}{\partial t} < 0 \Rightarrow \sigma < 0 \) (Compression)
Reflection and Transmission of Waves

Wave propagation at an interface

There are three waves at the interface:

- Incident $\sigma_I = f_I(x - c_1 t)$
- Reflected $\sigma_R = f_R(x + c_1 t)$
- Transmitted $\sigma_T = f_T(x - c_2 t)$

Force balance at $x = 0$:

$$\sigma_I\big|_{x=0^-} + \sigma_R\big|_{x=0^-} = \sigma_T\big|_{x=0^+}$$

$$f_I + f_R = f_T(*)$$

Velocity balance at $x = 0$:

$$\frac{\partial u_I}{\partial t}\big|_{x=0^-} + \frac{\partial u_R}{\partial t}\big|_{x=0^-} = \frac{\partial u_T}{\partial t}\big|_{x=0^+}$$

$$\frac{-f_I}{\rho_1 c_1} + \frac{f_R}{\rho_1 c_1} = -\frac{f_T}{\rho_2 c_2}(**)$$

From (*) and (**) we get,

$$f_R = \frac{\alpha - 1}{\alpha + 1} f_I$$

and

$$f_T = \frac{2\alpha}{\alpha + 1} f_I$$

where

$$\alpha = \frac{\rho_2 c_2}{\rho_1 c_1}$$

is the "acoustic impedance mismatch"
Reflection and Transmission of Waves

Free End Reflection:

- Incident $\sigma_I = f_I(x - c_1t)$
- Reflected $\sigma_R = f_R(x + c_1t)$

At the free end, stress is zero

$$\sigma|_{x=0} = f_I|_{x=0} + f_R|_{x=0} = 0 \implies f_R = -f_I$$

So, the pulse is of the same shape but of opposite sign, i.e., a compressive pulse reflects as a tensile pulse after interacting with a free surface and vice versa. What about particle velocity?

$$\frac{\partial u}{\partial t} = \frac{-f_I}{\rho_1 c_1} + \frac{f_R}{\rho_1 c_1} = \frac{-2f_I}{\rho_1 c_1}$$

The velocity doubles at the free surface
Reflection and Transmission of Waves

**Built in or Rigid End Reflection:**

- Incident $u_I = F_I(x - c_1 t)$
- Reflected $u_R = F_R(x + c_1 t)$

At a rigid end, the boundary conditions are $u = 0$ and $\dot{u} = 0$

$$u_{x=0} = F_I|_{x=0} + F_R|_{x=0} = 0 \Rightarrow F_R = -F_I$$

So, the velocity changes the sign in this case. What about stress?

$$\frac{\partial u}{\partial t} = \frac{-f_I}{\rho_1 c_1} + \frac{f_R}{\rho_1 c_1} = 0 \Rightarrow f_I = f_R \Rightarrow \sigma|_{x=0} = f_I + f_R = 2f_I$$

The stress doubles at a rigid end.
Reflection and Transmission of Waves

Method of Characteristics

Express the 1-D wave equation as \( c_0^2 \frac{\partial \varepsilon}{\partial x} - \frac{\partial \dot{u}}{\partial t} = 0 \) -(a)

Strain-displacement relationship gives us \( \frac{\partial \dot{u}}{\partial x} - \frac{\partial \varepsilon}{\partial t} = 0 \) -(b)

(a) + (b) \( \Rightarrow \) \( c_0(c_0 \varepsilon + \dot{u}),_x - (c_0 \varepsilon + \dot{u}),_t = 0 \) -(c)

Now, \( \frac{d(c \varepsilon + \dot{u})}{dt} = \frac{\partial (c \varepsilon + \dot{u})}{\partial x} \frac{dx}{dt} + \frac{\partial (c \varepsilon + \dot{u})}{\partial t} \)

If \( \frac{dx}{dt} = -c \), the above equation is zero and from (c) we get \( c \varepsilon + \dot{u} = \) constant. Similarly (a) - (b) will result in \( c \varepsilon - \dot{u} = 0 \) for \( dx/dt = c \).

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Stress wave is generated by launching a projectile onto the input bar.
Strains are measured in the input and the output bar.
Alignment of the bars and projectile are very important.
The bars need to move freely in the supports in their length direction.

A tensile pulse is generated by the sudden release of tensile strain stored in the bar using a clamp.

The pulse shape and wave form characteristics are influenced by the clamp design.

Finite element analysis is required for specimen design.

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3 Ramesh, Experimental Mechanics Handbook, 2009
Lagrangian or x-t diagrams are extremely useful in the analysis of Kolsky bar experiments.

4 Ramesh, Experimental Mechanics Handbook, 2009
In a torsion bar a torsional (shear) wave is propagated in the bars and the specimen.

- Torsional bars do not require dispersion correction.
- Large strains can be generated in a torsional bar.
- Bending waves need to be avoided during operation.
- Finite element analysis is required for the specimen design.

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*Ramesh, Experimental Mechanics Handbook, 2009*
Kolsky Bar Experiment - Torsion - x-t Diagram

ASM Metals Handbook, Vol. 8
Kolsky Bar Analysis - Compression and Tension

\[ \sigma(t) = \frac{P_1(t) + P_2(t)}{2A_0} \]

\[ P_1(t) = EA \varepsilon_1 = EA(\varepsilon_I(t) + \varepsilon_R(t)) \] and \[ P_2(t) = EA \varepsilon_2 = EA \varepsilon_T(t) \]

If the specimen stress state is homogeneous (equilibrium)

\[ P_1 = P_2 \] and \[ \varepsilon_I + \varepsilon_R = \varepsilon_T \]

Therefore, engineering stress and true stress are:

\[ \sigma(t) = \frac{EA \varepsilon_T(t)}{A_0} \] and \[ \sigma_t(t) = \sigma(t)[1 - \varepsilon(t)] \]
Kolsky Bar Experiments - Analysis

Compression and Tension

Now to calculate strain in the specimen, the average strain rate:

\[ \dot{\varepsilon}(t) = \frac{\dot{u}_2 - \dot{u}_1}{L_0} \]

\[ \dot{u}_1 = \dot{u}_I(t) + \dot{u}_R(t) = \frac{-\sigma_I}{\rho c_0} + \frac{\sigma_R}{\rho c_0} = \frac{-E \varepsilon_I + E \varepsilon_R}{\rho c_0} \quad \text{and} \quad \dot{u}_2 = \frac{-E \varepsilon_T}{\rho c_0} \]

But due to homogenization, \( \varepsilon_I + \varepsilon_R = \varepsilon_T \). So, engineering strain rate and true strain rate are given by

\[ \dot{\varepsilon} = \frac{2E \varepsilon_R(t)}{\rho c_0 L_0} = \frac{2c_0 \varepsilon_R(t)}{L_0} \quad \text{and} \quad \dot{\varepsilon}(t) = \frac{\dot{\varepsilon}(t)}{1 - \varepsilon(t)} \]

Then the engineering strain and true strain are calculated using,

\[ \varepsilon(t) = \int_0^t \dot{\varepsilon}(\tau) d\tau \quad \text{and} \quad \varepsilon(t) = -\ln[1 - \varepsilon(t)] \]
Kolsky Bar Experiments - Compression Analysis

\[ \text{Input bar signal} \]

\[ \text{Output bar signal} \]

\[ Ramesh, \text{ Experimental Mechanics Handbook, 2009} \]
Kolsky Bar Experiments - Compression Analysis

Vanadium (600 °C) at 4500 s⁻¹

True stress (MPa)

0 0.05 0.1 0.15 0.2 0.25

0 50 100 150 200 250 300

Vanadium (600 °C) at 4500 s⁻¹

Kolsky Bar Experiments - Torsion Bar Analysis

The torque, $T$ and the angular velocity, $\dot{\theta}$, of the bar for a given density $\rho$ and polar moment of inertia $J_c$ are related by:

$$T = \rho J_c \dot{\theta}$$

Maximum torque in a bar is given by:

$$T_{\text{max}} = \frac{\tau_y J_c}{r} \Rightarrow \dot{\theta} = \frac{\tau_y}{r \sqrt{\rho G}}$$

The average shear strain rate $\dot{\gamma}_s(t)$ and shear strain are given by:

$$\dot{\gamma}_s(t) = \frac{r_s}{L_s} [\dot{\theta}_1 - \dot{\theta}_2] \text{ and } \gamma_s(t) = \int_0^t \dot{\gamma}_s(\tau) d\tau$$

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Kolsky Bar Experiments - Torsion Bar Analysis

The relative change in angular velocity is given by (with one strain gage):

\[
[\dot{\theta}_1 - \dot{\theta}_2] = \frac{2[-T_R(t)]}{\rho J_c} \quad \text{and} \quad T_R(t) = \frac{GJ_c \gamma_R(t)}{r_b}
\]

Now to compute the shear stress we use to the strain measured in the transmitted bar, \( \gamma_T \) assuming that the stress in the specimen is uniform \( \Rightarrow \) torque at both ends of the specimen is same

\[
\tau_s(t) = \frac{T_s(t)}{2\pi r_s^2 t_s} = \frac{Gr_b^3}{4r_s^2 t_s} \gamma_T(t)
\]

where, \( r_s \) is the mean radius of an hollow specimen, \( r_b \) is the radius of the torsion bars, \( t_s \) is the thickness of the specimen, \( T_R(t) \) torque calculated from the reflected strain. ¹⁰

¹⁰ ASM Metals Handbook, Vol 8
In the torsional Kolsky bar experiments static compression and tension loads can be superimposed.

Due to the long pulses generated in the torsional bars the bending and axial forces need to be avoided.

Specimen gripping is critical as any misfit arrangement can cause long rise times and errors in strain measurement.

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ASM Metals Handbook, Vol 8
Kolsky Bar Experiments - Adiabatic Heating

- Under dynamic loading, inelastic deformation causes temperature rise in the specimen.
- The heat removal rate in a material depends on the thermal diffusivity $\alpha$ ($= 1.14 \text{cm}^2\text{s}^{-1}$ for copper).
- The distance the heat travels is given by $2\sqrt{\alpha t}$.
- At higher strain rates, the heat generated from the specimen during entire experiment is accumulated - this is called adiabatic heating.
- Temperature rise during adiabatic heating is given by

$$\Delta T = \frac{\beta}{\rho C_p} \int_{0}^{\varepsilon_f} \sigma d\varepsilon$$

where $\beta$ depends on the plastic work converted into heat.

- Implications of this adiabatic heating include thermal softening and shear banding.
Kolsky Bar Experiment - Design

- Kolsky bar design has three important ratios to consider but most importantly the specimen size, i.e., for a cylindrical sample the length $l_0$ and diameter $d_0$ and the projectile velocity $V$ are chosen first.

- The ratios are:
  1. $L/D$ - Bar length to bar diameter ratio - typically about 100
  2. $D/d_0$ - Bar diameter to specimen diameter - about 2 to 4
  3. $l_0/d_0$ - Specimen length to its diameter - about 0.5 to 1

- The main components of Kolsky bar setup are:
  1. Projectile launcher - e.g., compressed air gas gun
  2. Input and output bars - e.g., maraging steel bars
  3. Strain gauges to measure strain in the bars - e.g., foil gages
  4. Alignment fixture to keep the bars straight and concentric
  5. Data acquisition with required bandwidth and resolution - e.g., oscilloscope

- The specimen size chosen should represent the bulk response of the materials.
Kolsky Bar Experiment - Gas Gun

Design parameters include:
- Projectile mass
- Projectile velocity
- Pressure in the breach chamber
Kolsky Bar Experiment - Strain Measurement

- Quarter Bridge vs. Half Bridge - Depends on length of the bar and output signal required
- Bonded Foil vs. Semiconductor Strain Gages - Depends on gage factor required
- Signal Amplification is necessary when amplitude is small
- Data acquisition should have enough bandwidth to capture sharp gradients in the signal
High Strain Rate Experiments

Kolsky Bar Experiment - Specimen Geometry

Considerations for specimen geometry and preparation

- Specimen surfaces need to be flat, parallel and smooth for compression experiments
- Specimen should not have stress concentration regions for all compression, tension and torsion
- In the case of tension and torsion finite element calculations need to be performed with the end conditions
- While testing ceramic specimens the punch effect should be avoided
- For ceramic specimens pulse shaping may be required to allow uniform loading before critical strength is reached
- For very soft materials lower stiffness bars are required to measure stress
- For tension and torsion experiments local strain measurement may be required for accurate stress vs. strain curves
- Strain control by varying pulse width or by using a collar
Kolsky Bar Experiment - Requirements

- **Force equilibrium** ⇒ $P_1 = P_2$
- Stress equilibration is required for output bar measurement to represent average stress in the specimen.
- **Friction Effects** - will lead to specimen barreling and uniaxial stress state is not maintained
- Friction can be removed by ensuring flat and smooth bar and specimen surfaces and by the use of lubricants
- **Dispersion Effects** - Longitudinal waves suffer from geometric dispersion, e.g., due to Poisson expansion or contraction.
- Dispersion effects the strain pulses, superimposes oscillations in the specimen loading and shorter pulses cause much worse dispersion effects.
- Dispersion effects can be removed by corrections performed on the strain signals or by introducing pulse shaping
Desktop Kolsky bar designed to load specimens at very high strain rates of $10^4$ to $10^5$

The typical projectile velocity ranges from 10-50 m/s

Jonnalagadda et al., MSE-A, submitted
Dynamic Strength of Armor Materials

- Dynamic strength, failure, rate-dependence and fracture in metals and ceramics
- Temperature-Strain Rate dependence in materials
- Phenomelogical and physical models such as Johnson & Cook, Zerilli-Armstrong (Z-A) and Mechanical Threshold Stress (MTS) for material behavior


